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PART D: ALGEBRA

INTRODUCTION

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Part D contains discussions by leading mathematicians of algebraic ideas that have strongly influenced the development of modern mathematics. It begins with a broad survey by Jean Dieudonné, a prime mover in the creation of the Bourbaki encyclopedia [11], whose systematic organization and lucid style mesmerized a whole generation of American graduate students.

In his survey, Dieudonné shows how many of the most important mathematical developments of our century have evolved from the imaginative "fusion" of two or more different topics. A typical example was the fusion of Diophantine equations, whose problems form a "center of attraction", with geometry, a "center of radiation". This survey is interesting not only for the insight that it gives into the way in which mathematics develops, but perhaps even more as a collection of thumbnail sketches of historical interpretations, waiting to be expanded into substantial accounts by later historians. It stimulated a lively if somewhat disjointed discussion, mostly about questions and viewpoints not covered in Prof. Dieudonné's talk.

The second paper is by Alan Baker, and constitutes a case study of progress since Euler on Catalan's problem of finding all pairs of successive integers that are powers of smaller integers--probably 8 and 9 are the only ones! Baker shows how this "center of attraction" stimulated and guided much good mathematics. This seems to be a characteristic feature of difficult "elementary" problems in number theory. The discussion of Baker's paper continued the exchange of opinions about the nature of mathematical progress which began after Dieudonné's paper.

Next comes George Mackey's paper, which shows how special methods, discovered independently in apparently different contexts, are often recognized much later as just different instances of the same general theory. For example, this is true of harmonic analysis. Theorems about functions and characters of general locally compact groups contain as special cases much of the adele theory discussed by Dieudonné, as well as number theory and parts of quantum mechanics. A technical discussion followed this paper, in which various *analytic* aspects of harmonic analysis were brought out (cf. Part E) that may be lost sight of in thinking about its algebraic aspects.

The last two papers are of a much more historical nature. In the first, Thomas Hawkins cites four examples from 19th century mathematics in which progress did not result from fusion in Dieudonné's sense. The second, by Shreeram Abhyankar, expands on critical comments he made in discussion concerning the Bourbakist interpretation of progress in algebraic geometry. He argues that the sophistication imposed by modern algebraic terminology actually obscures the *real* progress, which comes from new algebraic algorithms similar to those taught in high-school.